

Practice Exam 3
Sections 3.3-3.4 and 4.1-4.3

Treat this like an exam. Answer the following questions. *You must show your work to receive full credit.*

1a. . Jim buys a car. Every year, Jims car depreciates (loses) 12% of its value from the previous year. When the car is new (year 0), it is valued at \$26,000. Give a recurrence relation for the value of Jims car after n years for $n \geq 0$. You must use correct notation.

1b. How many ways are there to rearrange the word SCARF?

1c. How many ways are there to rearrange the word MISSISSIPPI?

2. A committee is made up of 7 women and 4 men.

- (a) How many ways are there to form a subcommittee of 3 people?
- (b) How many ways are there to form a subcommittee of 3 people if the subcommittee must contain at least 2 women?
- (c) How many different linear seating arrangements of the committee are there?
- (d) How many different linear seating arrangements of the committee are there if the 4 men must sit together?
- (e) How many different linear seating arrangements of the committee are there if none of the 4 men are allowed to sit together?

3. The following problem refers to strings from the English alphabet $\{A, B, \dots, Z\}$.

- (a) How many 4-letter strings are possible?
- (b) How many 4-letter strings are possible if no letter can be repeated?
- (c) How many 4-letter strings are there that end with the letter B?
- (d) How many 4-letter strings are there with exactly one B?

4. Prove Theorem 4.1 from the text. (This is Theorem 1 from Worksheet Section 4.1.)

5. Prove Theorem 4.2 from the text. (This is Theorem 2 from Worksheet Section 4.1.)

6. In the following suppose that a *license plate* is made up of 6 entries from the alphabet $\{A, B, \dots, Z\} \cup \{0, 1, \dots, 9\}$.

- (a) How many license plates are there which have 3 letters followed by 3 numbers?
- (b) How many license plates are there which contain exactly 4 letters?
- (c) How many palindromic license plates are there?
- (d) How many palindromic license plates are there with 4 numbers?

7a. Suppose that $f : X \rightarrow Y$ is a seven-to-one function mapping X onto Y and that $|Y| = 6$. What is $|X|$? Give a brief explanation of your answer. If there is not enough information, state so and explain why.

7b. Suppose that $f : X \rightarrow Y$ is a one-to-one correspondence and that $|X| = 8$. What is $|Y|$? Give a brief explanation of your answer. If there is not enough information, state so and explain why.

7c. Suppose that $f : X \rightarrow Y$ is a two-to-one function and that $|X| = 8$. What is $|Y|$? Give a brief explanation of your answer. If there is not enough information, state so and explain why.

8a. Explain why, in a class of 32, there will always be a group of at least 5 students who were born on the same day of the week.

8b. Explain why for every 27-word sequence in the US Constitution, at least two words will start with the same letter.